IPS 2024

IPS 24-4.2

On the measurement of permeability anisotropy and associated error: application to API RP 19B Section 4 tests

Presented by:
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HALLIBURTON Wireline & Perforating

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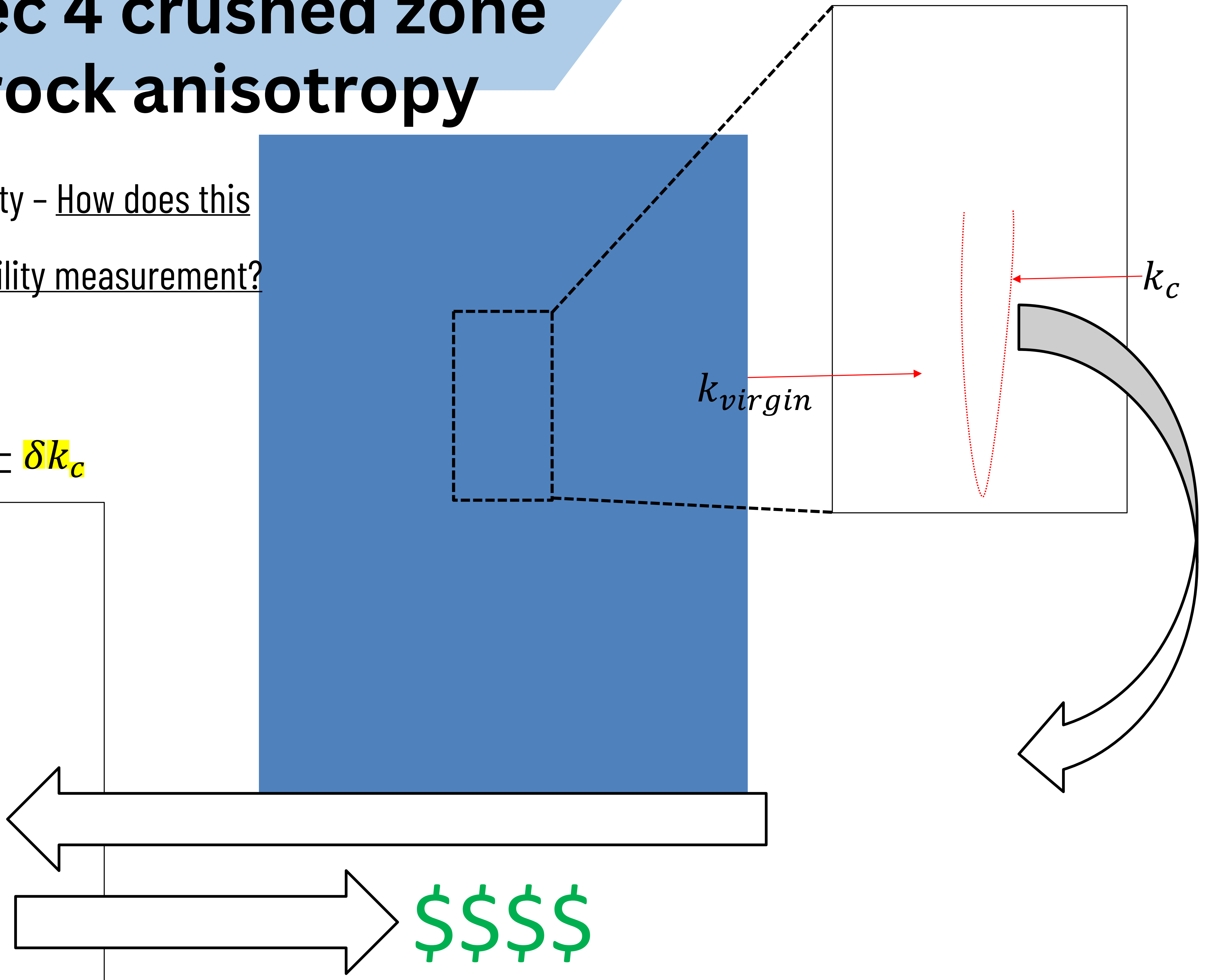
Background: error in Sec 4 crushed zone permeability due to rock anisotropy

- Rock is anisotropic with respect to permeability – How does this influence the accuracy of a k_{virgin} permeability measurement?
- API RP 19B Section 4 Perforate-and-Flow Test:

$$k_{virgin} \pm \delta k_{virgin} \rightarrow k_c \pm \delta k_c$$



Background: theoretical rate index, RI_{th}

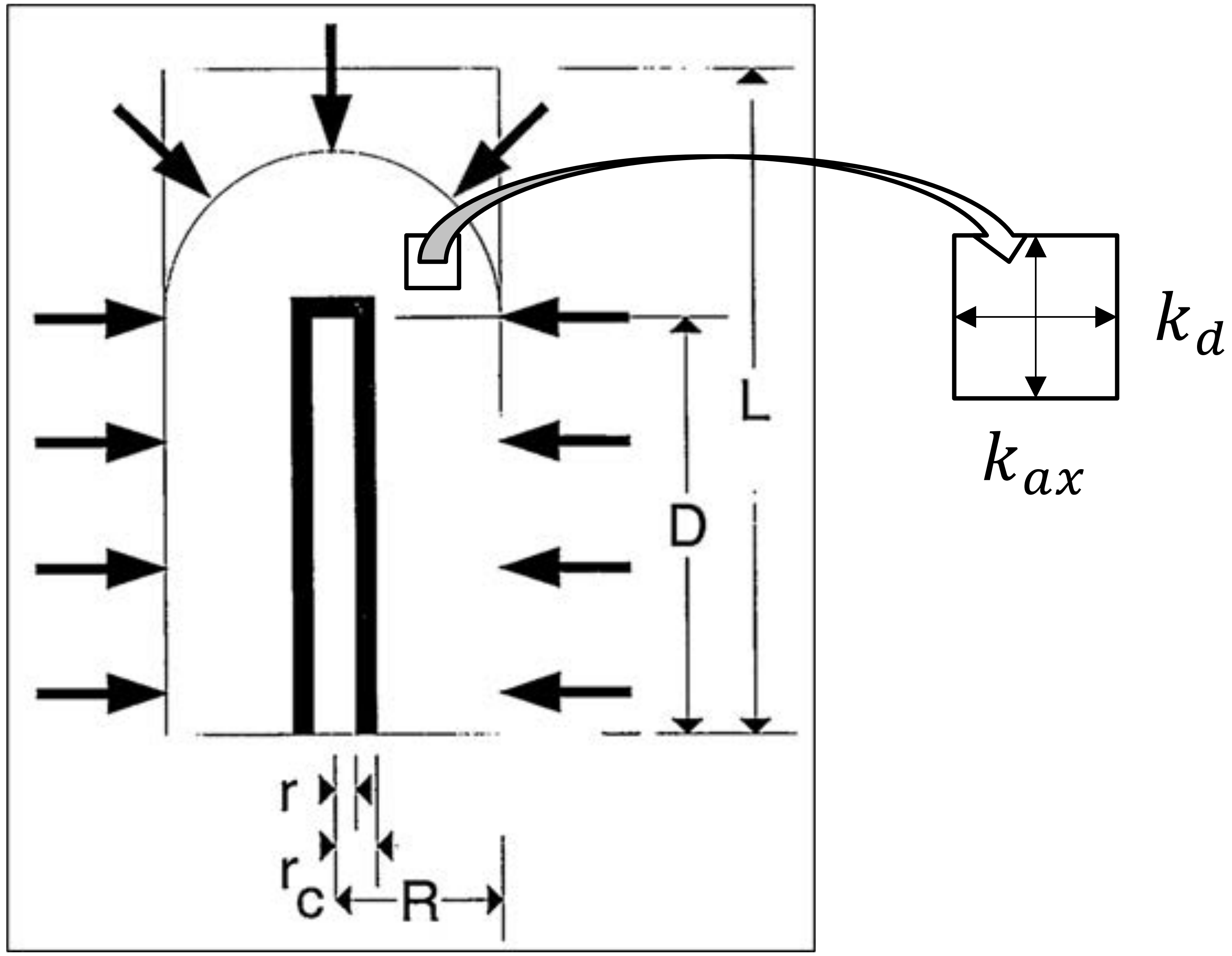
API RP 19B, Section 4:

$$RI_{th} = \frac{2\pi}{\mu}\left[\frac{k_d D}{\ln(R/r_c)} + \sqrt[3]{k_{ax}k_d^2}\left(\frac{r_c R}{R - r_c}\right)\right]$$

k_{ax} = "axial permeability"; k_d= "diametral permeability"

$$\rho\left(\frac{\partial v_i}{\partial t} + v_j\frac{\partial v_i}{\partial x_j}\right) = -\frac{\partial p}{\partial x_i} + \mu\frac{\partial^2 v_i}{\partial x_j \partial x_j} + \rho g_i + S_i$$

$$S_i = -\left(r_{ij}\mu v_j + \rho\beta_{ij}|v|v_j\right);\ r_{ij} = k_{ij}^{-1}$$



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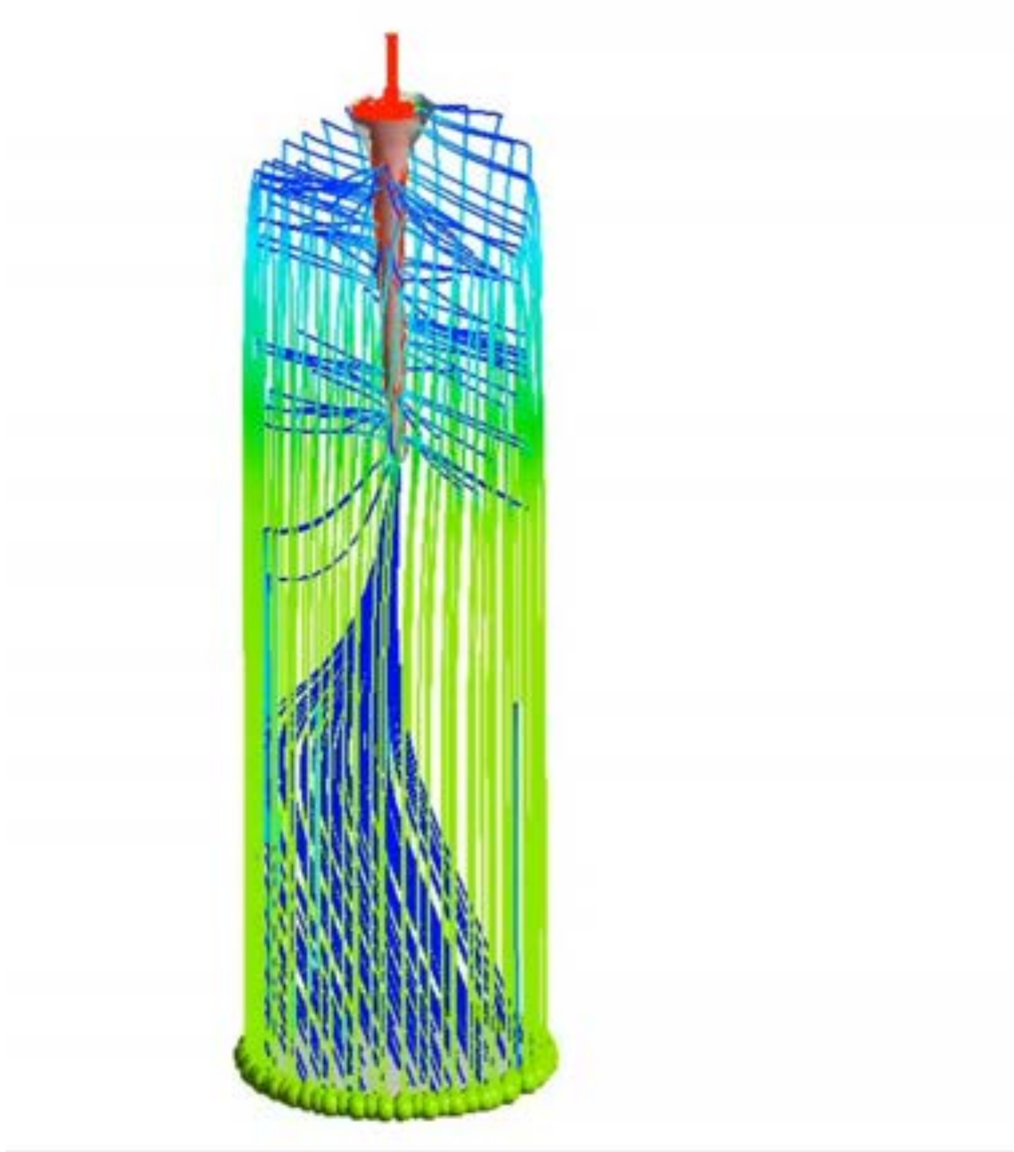


Background: permeability and viscous resistivity

Darcy's law (permeability):

$$q_i = -\mu^{-1} k_{ij} \partial\Phi/\partial x_j$$

Darcy's law (viscous resistivity)

$$-\partial\Phi/\partial x_i = \mu r_{ij} q_j$$

$$r_{ij} = k_{ij}^{-1}$$

$$r_{ij} = \frac{\text{cofactor of } k_{ij}}{\det k_{ij}}$$

$$\text{cofactor of } k_{ij} = \epsilon_{ikl}\epsilon_{jmn}k_{mk}k_{nl}/2$$

$$\det k_{ij} = \epsilon_{ijk}k_{1i}k_{2j}k_{3k}$$

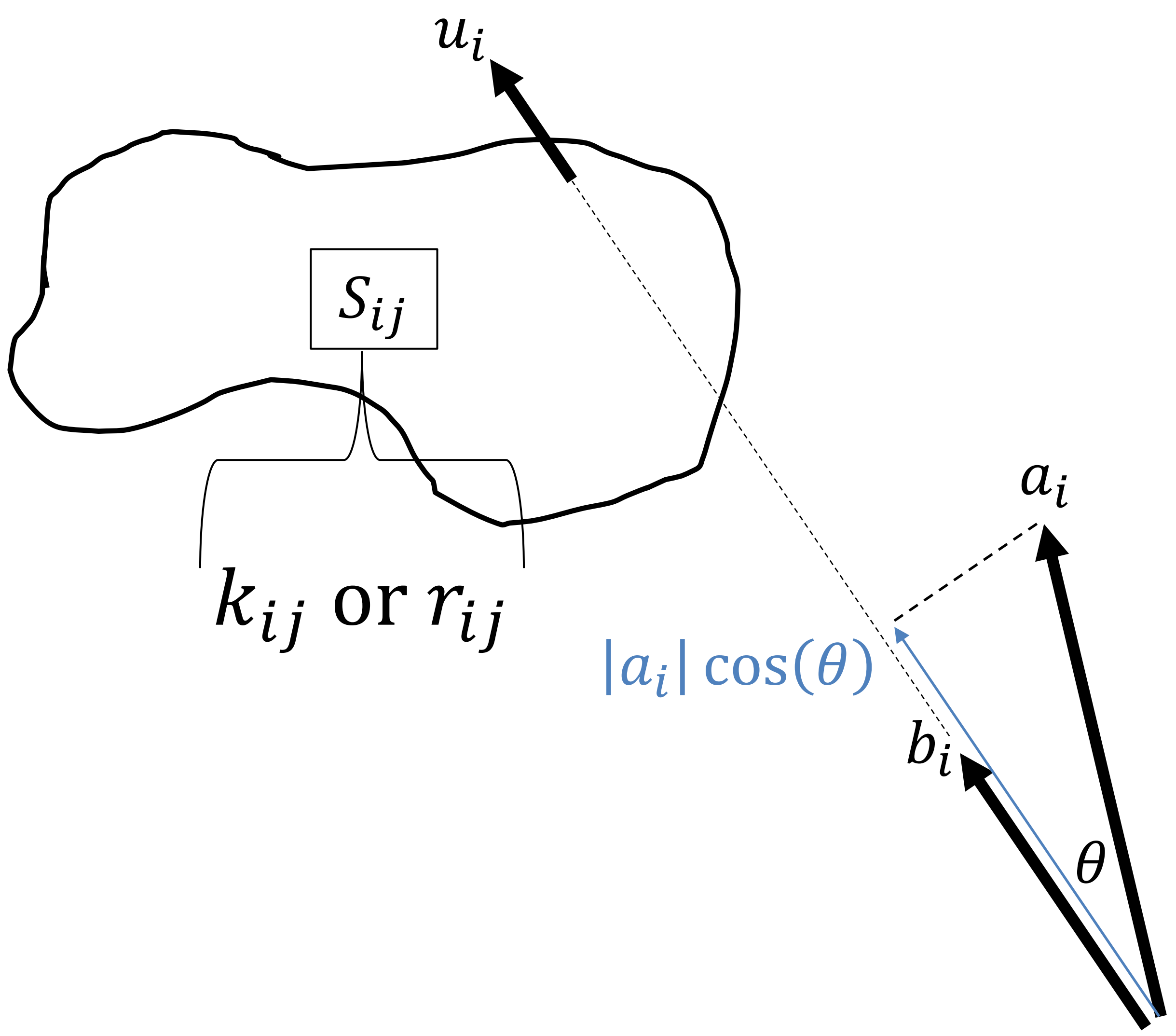
Background: magnitude of symmetric 2nd-rank tensor

$$S_{ij} = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}; S_{ij} = S_{ji}$$

$$S = S_{ij} u_i u_j \quad (1)$$

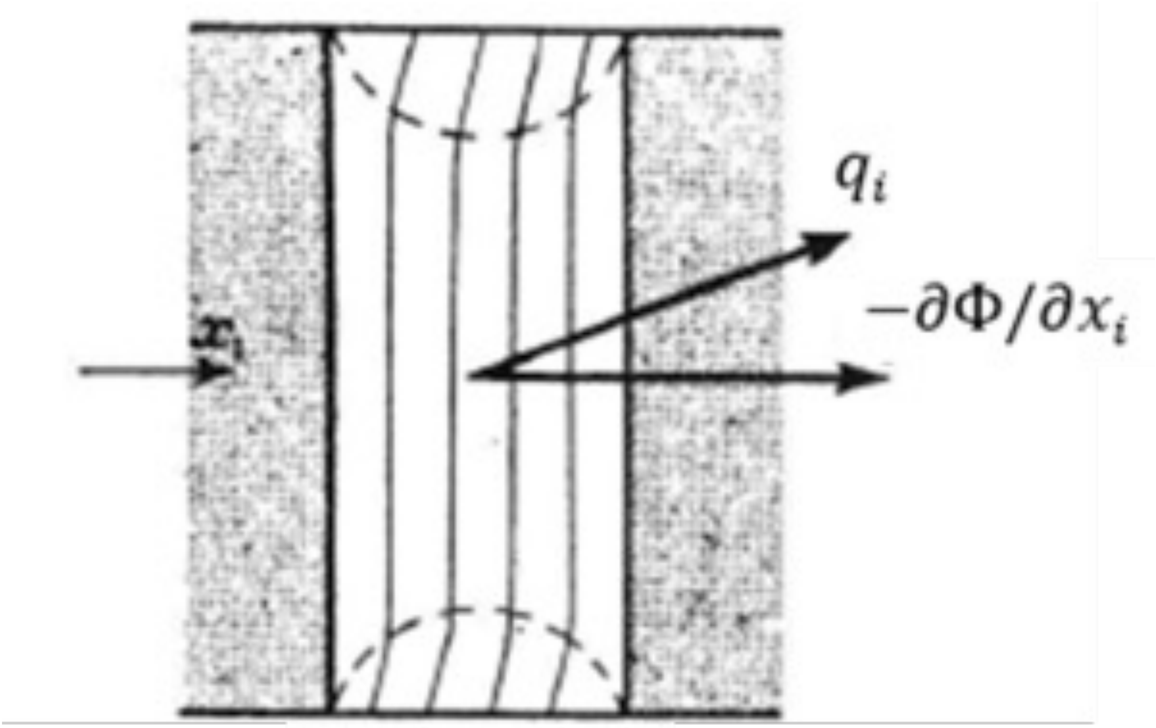
$$a_i = S_{ij} b_j \quad (2)$$

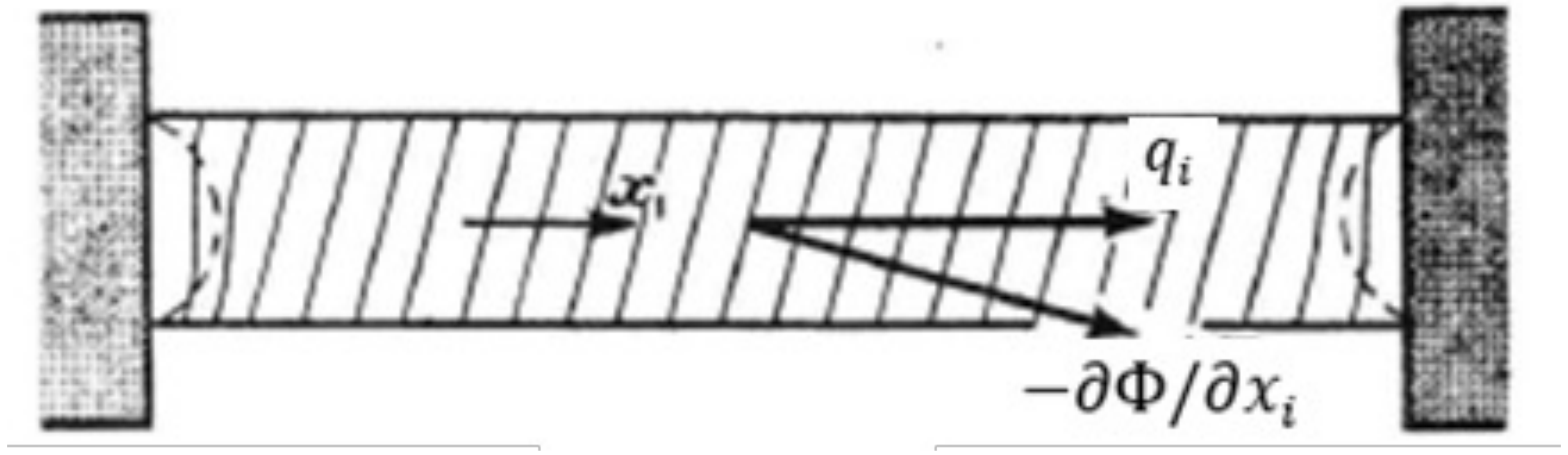
$$S = \frac{|a_i| \cos(\theta)}{|b_i|} \quad (3)$$



Background: experimental measurement

$$S = |a_i| \cos(\theta) / |b_i| \quad (3)$$





- Thin-disc, $D/L \gg 1$
- Vector $-\partial\Phi/\partial x_i$ forced along x_1

$$q_i = -\mu^{-1} k_{i1} \partial\Phi/\partial x_1$$

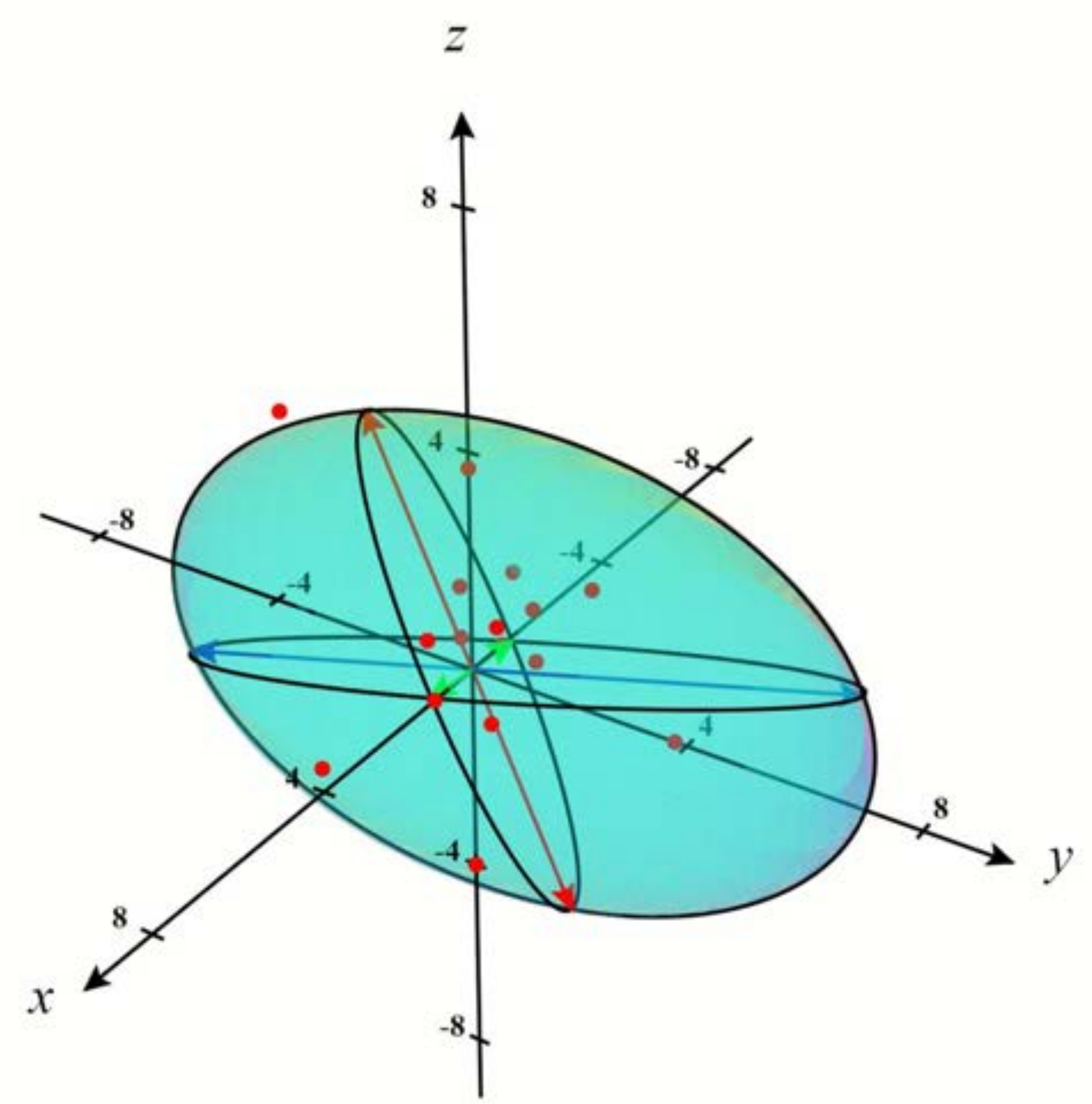
- From Eq(3), measure q_1 to determine k_{11}

- Long rod, $D/L \ll 1$
- Vector q_i forced along x_1

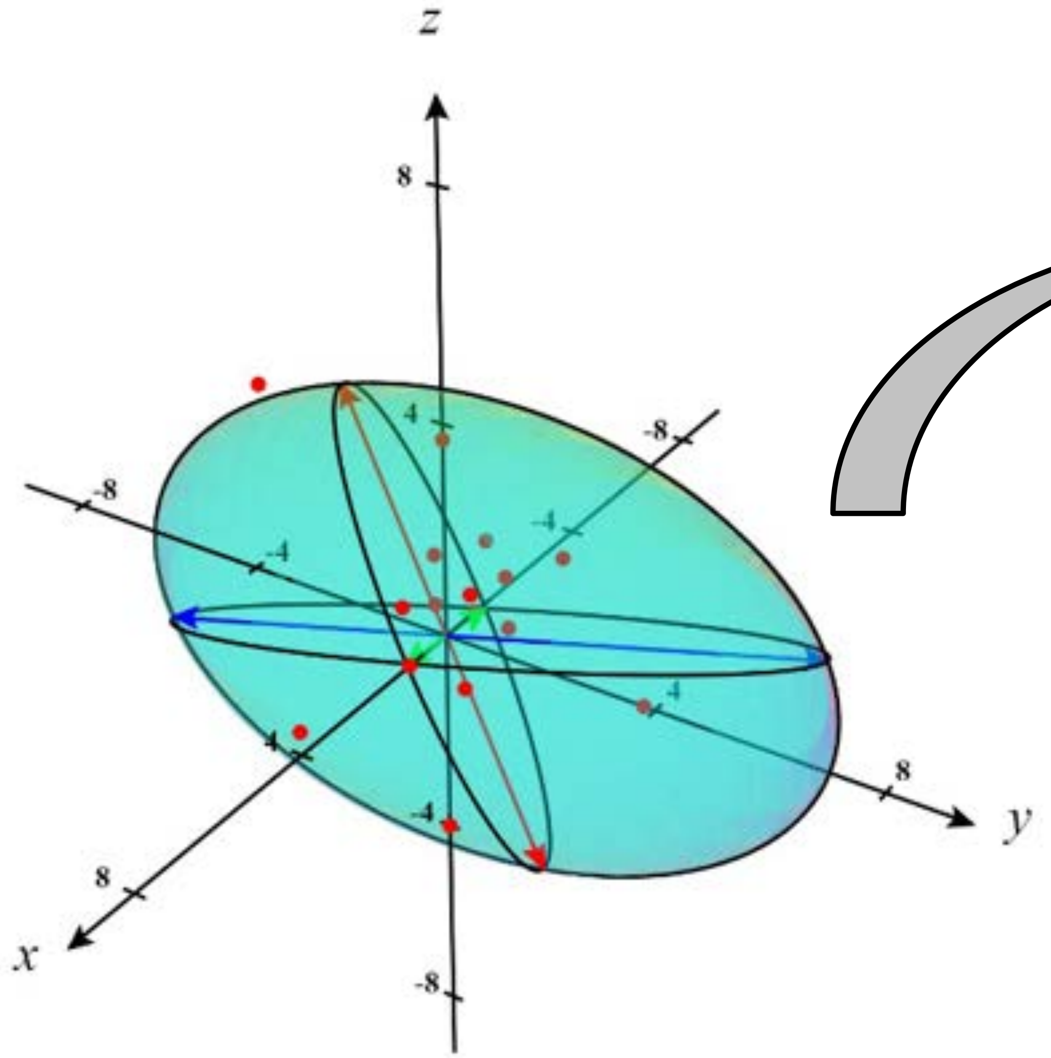
$$-\partial\Phi/\partial x_i = \mu r_{i1} q_1$$

- From Eq(3), measure $-\partial\Phi/\partial x_1$ to determine r_{11}

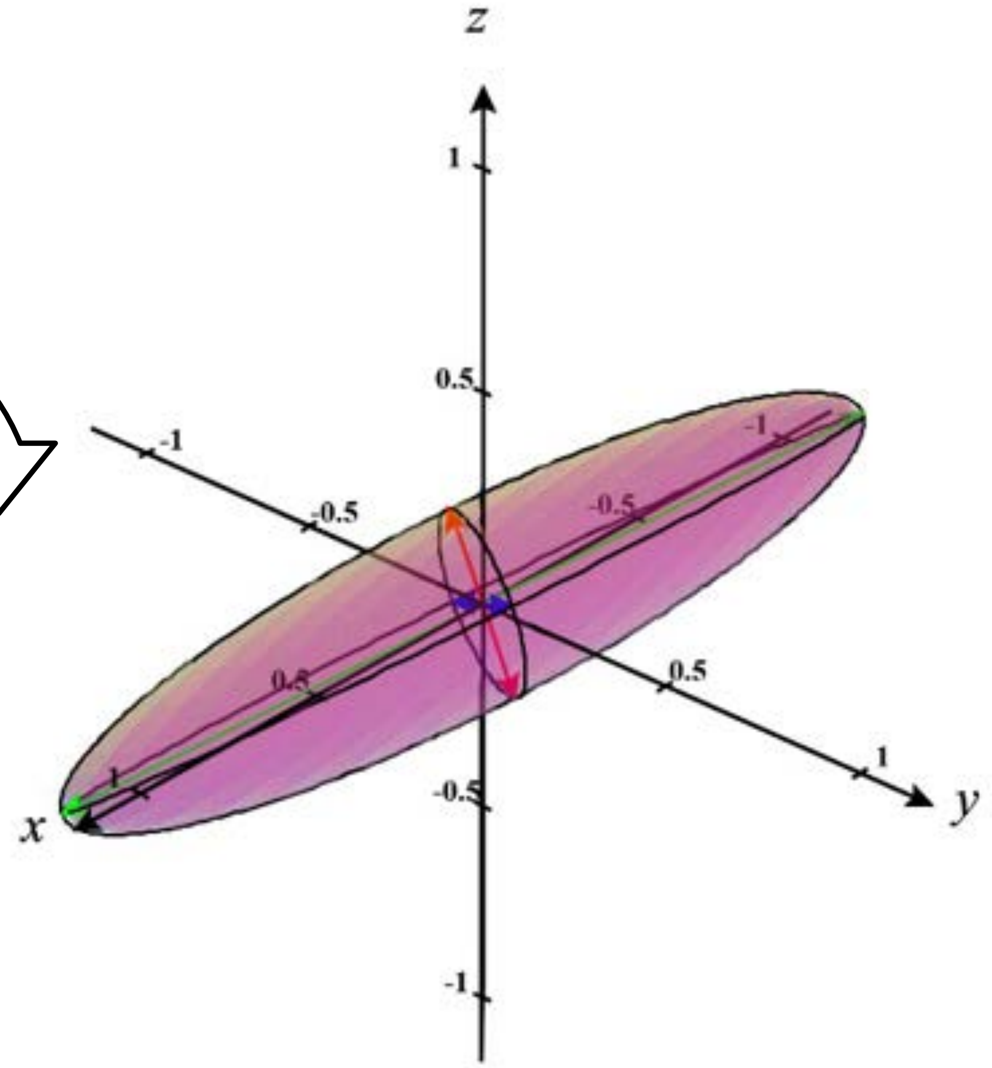
Background: experimental measurement



Background: experimental measurement



$$k_{ij} = r_{ij}^{-1}$$



$$r_{ij}(x,y,z) = \begin{bmatrix} 1.07 & -0.20 & -0.12 \\ -0.20 & 0.06 & 0.01 \\ -0.12 & 0.01 & 0.05 \end{bmatrix} \text{md}^{-1}$$

$$k_{ij}(x,y,z) = \begin{bmatrix} 4 & 13 & 6 \\ 13 & 62 & 17 \\ 6 & 17 & 29 \end{bmatrix} \text{md}$$

$$r'_{ij}(x',y',z') = \begin{bmatrix} 1.12 & 0 & 0 \\ 0 & 0.05 & 0 \\ 0 & 0 & 0.014 \end{bmatrix} \text{md}^{-1}$$

$$k'_{ij}(x',y',z') = \begin{bmatrix} 0.09 & 0 & 0 \\ 0 & 21.5 & 0 \\ 0 & 0 & 72.7 \end{bmatrix} \text{md}$$

Questions

What dimensions does the sample need to have to yield k with a certain desired accuracy or, more generally, what is the relation between the geometry of a cylindrical sample, expressed by the D/L ratio, and the apparent directional permeability k_a for a given anisotropy k_{ij} and measurement orientation u_i? The same can be asked for r.

Note, for finite-dimensioned samples, apparent directional k or r in the u_i direction measured using a conventional axial permeameter :

- $k_a = \frac{Q\mu L}{A\Delta\Phi}$
- $r_a = \frac{A\Delta\Phi}{Q\mu L}$

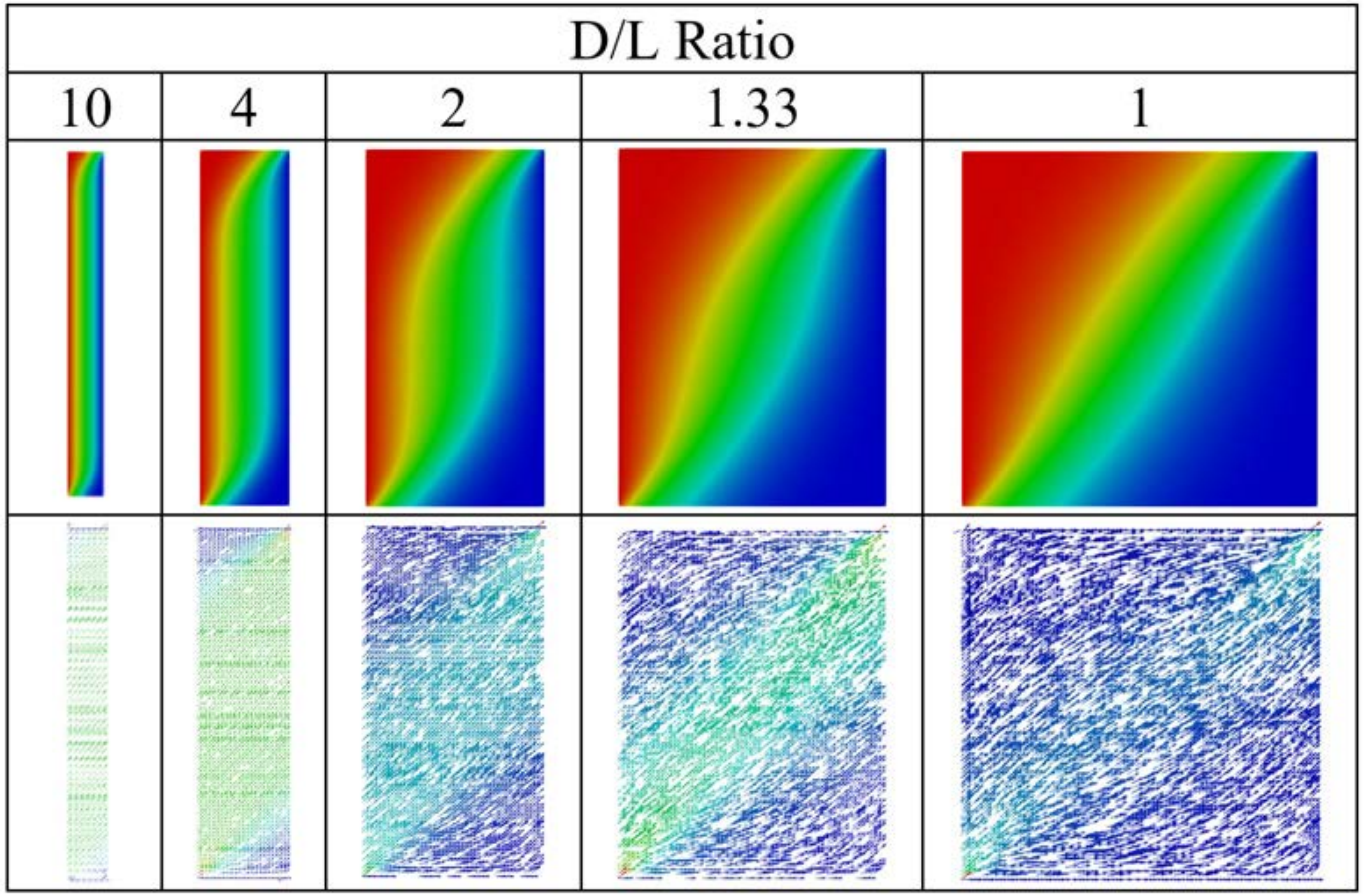
Numerical model

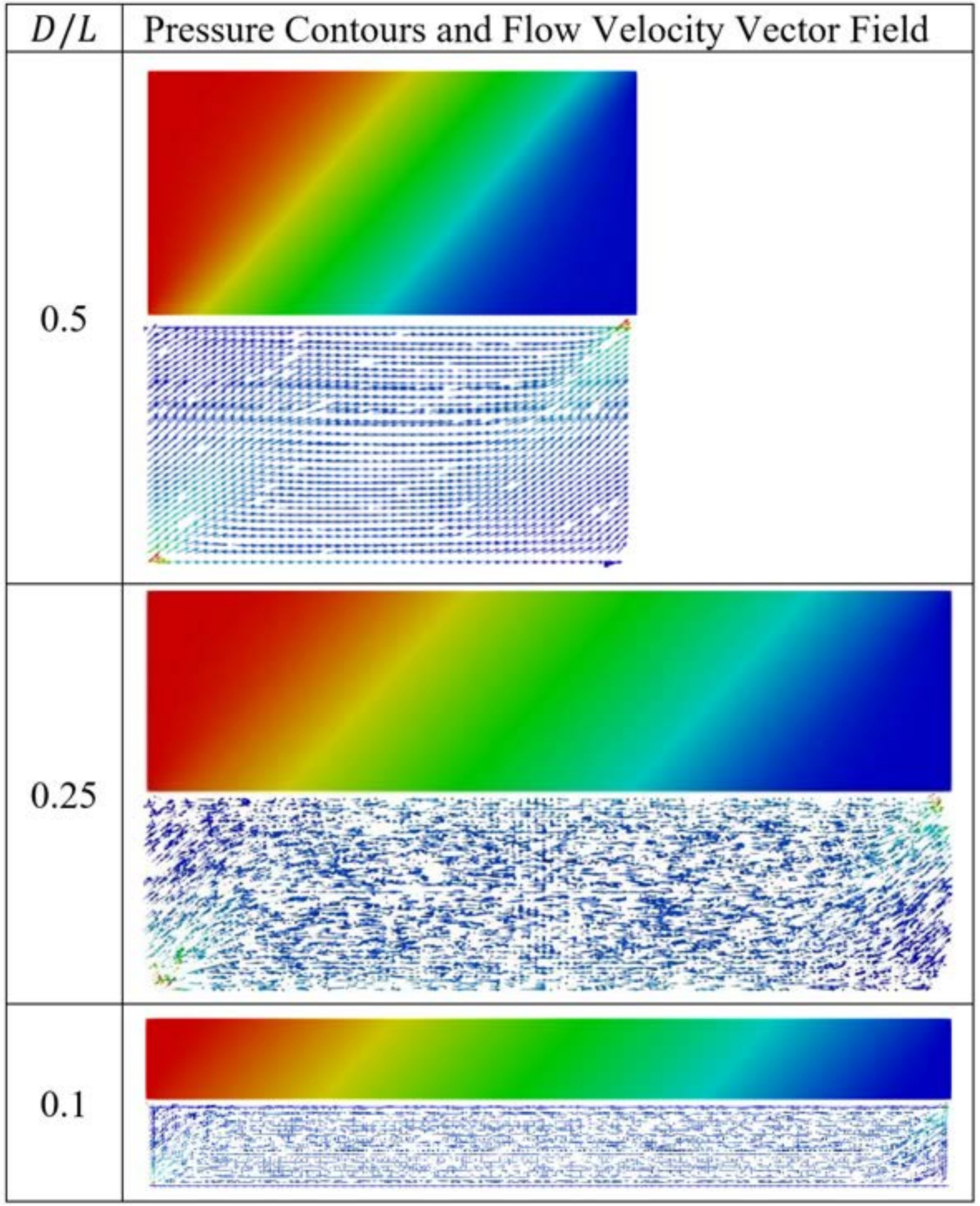
$$k'_{ij}\ (x', y', z') = \begin{bmatrix} 20 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{md}$$

True k_{xx}

$$k_{ij}\ (x, y, z) = \begin{bmatrix} 10.5 & 9.5 & 0 \\ 9.5 & 10.5 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{md}$$

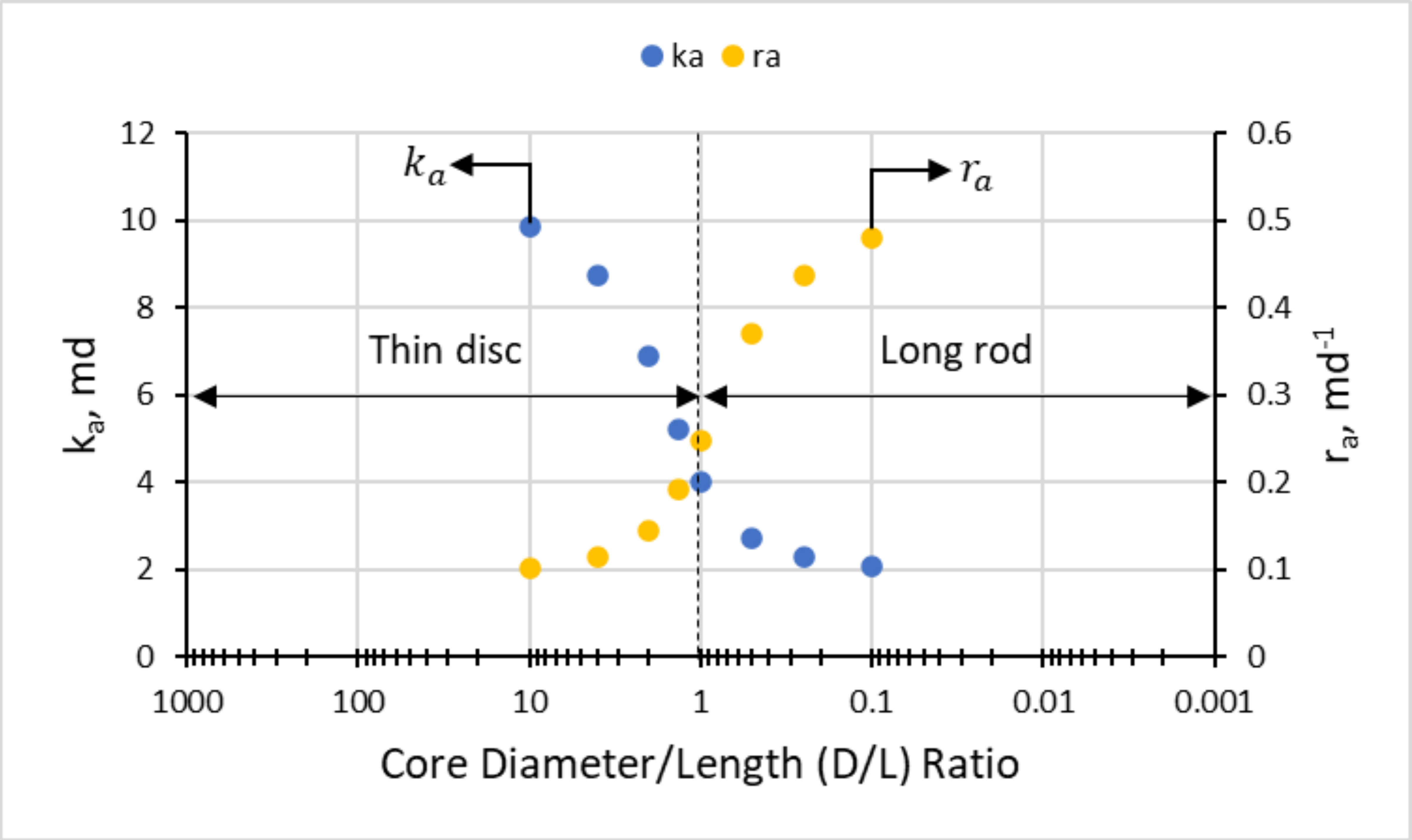
Simulation results





Simulation results

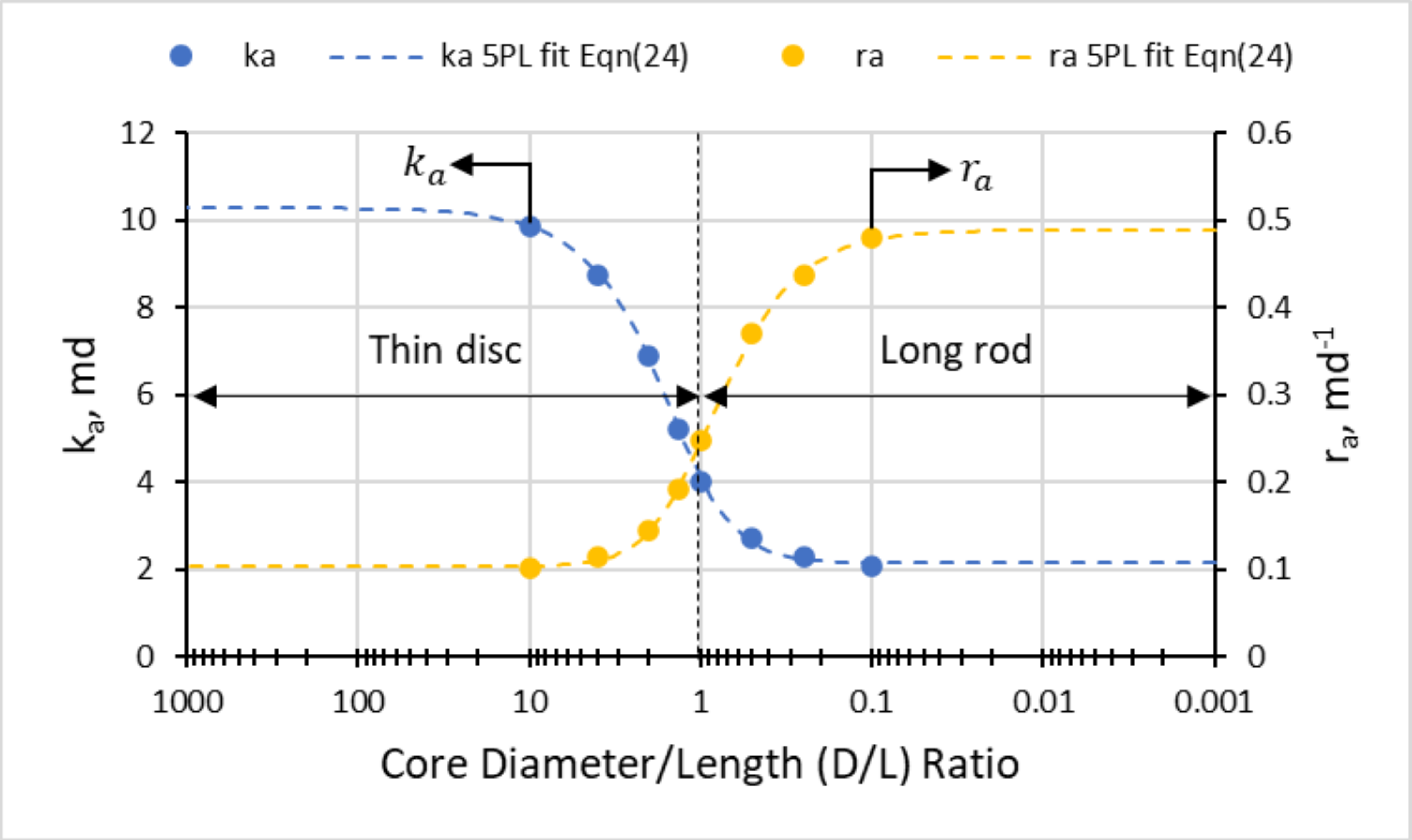
- $k_a = \frac{Q\mu L}{A\Delta\Phi}$
- $r_a = \frac{A\Delta\Phi}{Q\mu L}$



Simulation results

- $k_a = \frac{Q\mu L}{A\Delta\Phi}$
- $r_a = \frac{A\Delta\Phi}{Q\mu L}$
- $y = d + \frac{(a-d)}{\left(1+(x/c)^b\right)^g}$ (24)

5PL Curve Fit		
Parameter	k_a	r_a
a	2.16	0.49
b	2.52	1.60
c	1.18	1.46
d	10.28	0.10
g	0.54	2.30
SSE	0.033	7.98e-5



Simulation results

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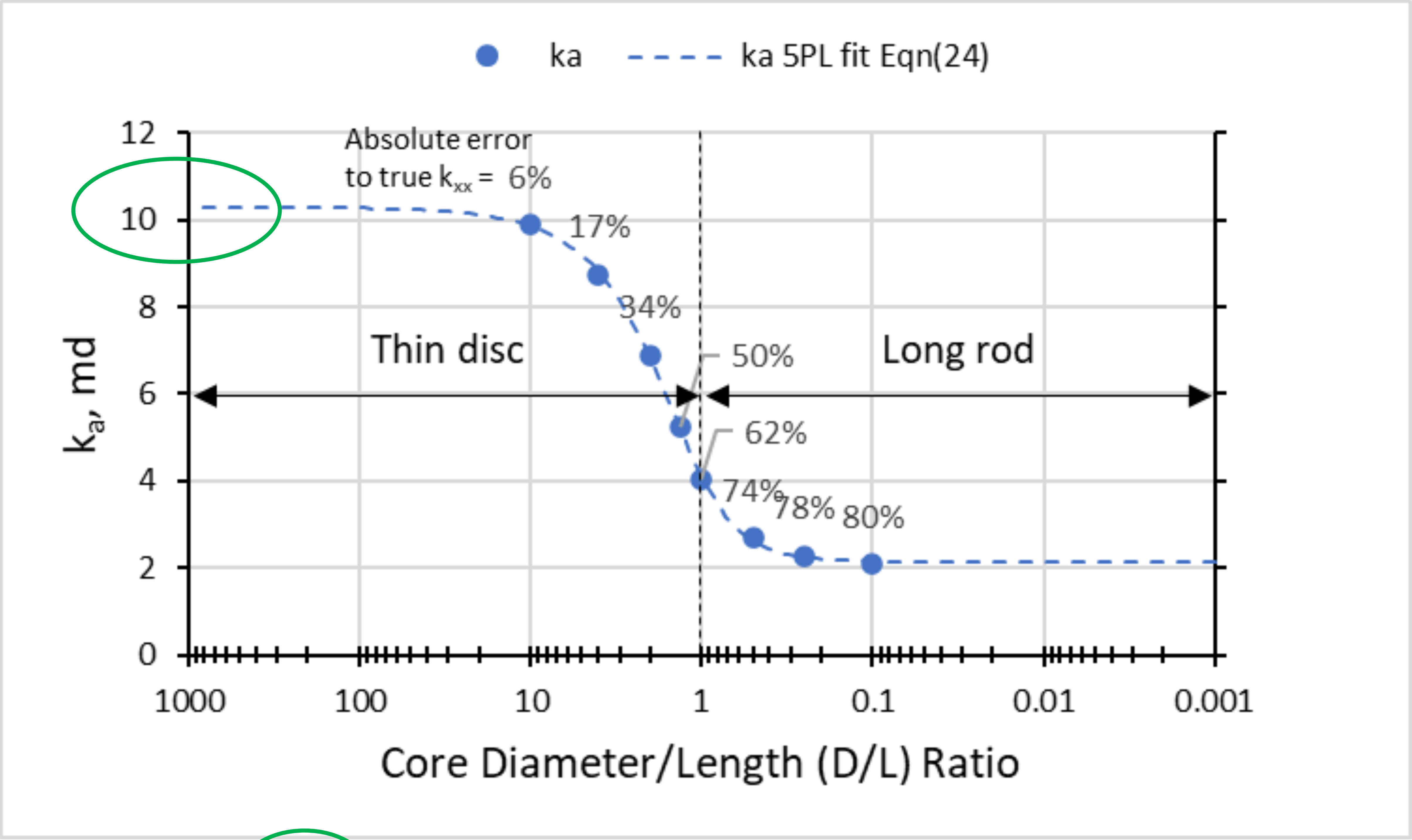

$$k_{ij}(x,y,z) = \begin{bmatrix} 10.5 & 9.5 & 0 \\ 9.5 & 10.5 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{md}$$

Simulation results

- $k_a = \frac{Q\mu L}{A\Delta\Phi}$
- $r_a = \frac{A\Delta\Phi}{Q\mu L}$
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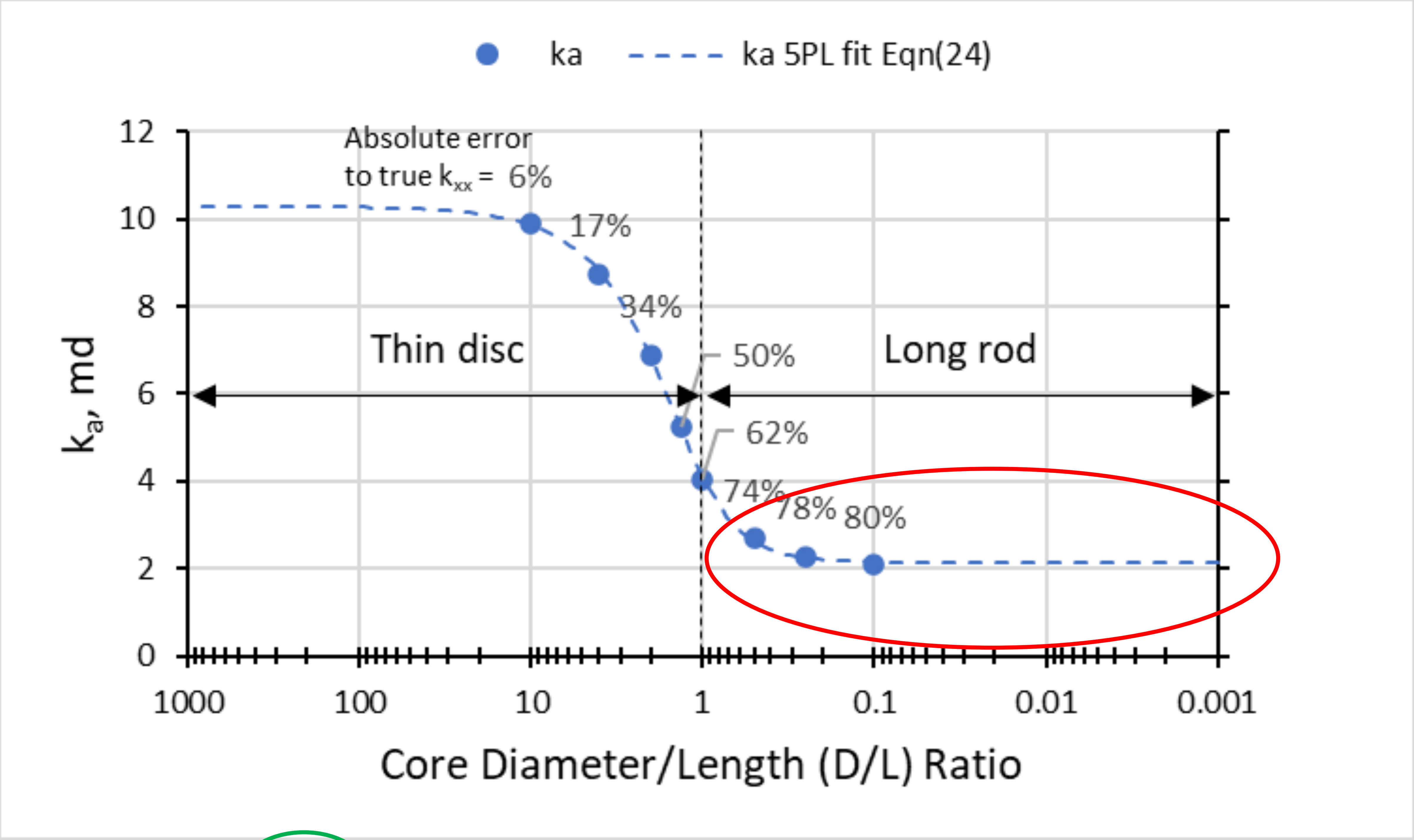

$$k_{ij}(x,y,z) = \begin{bmatrix} 10.5 & 9.5 & 0 \\ 9.5 & 10.5 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{md}$$

Simulation results

- $k_a = \frac{Q\mu L}{A\Delta\Phi}$
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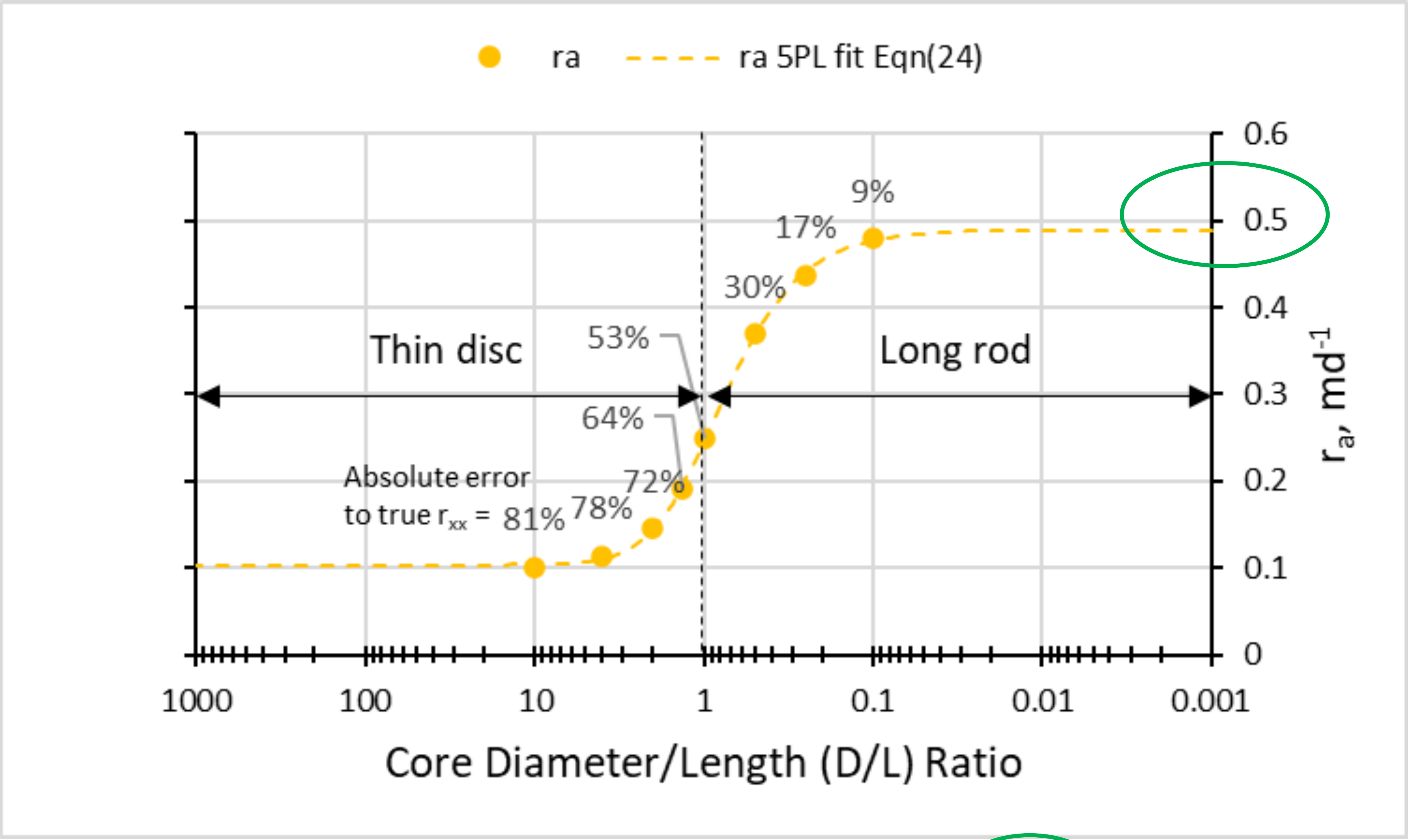
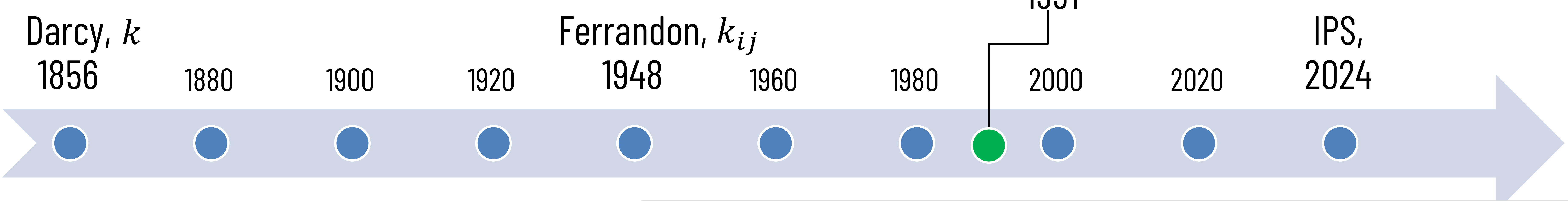


$$k_{ij}(x,y,z) = \begin{bmatrix} 10.5 & 9.5 & 0 \\ 9.5 & 10.5 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{md} \qquad r_{ij}(x,y,z) = \begin{bmatrix} 0.525 & -0.475 & 0 \\ -0.475 & 0.525 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{md}^{-1}$$

Simulation results

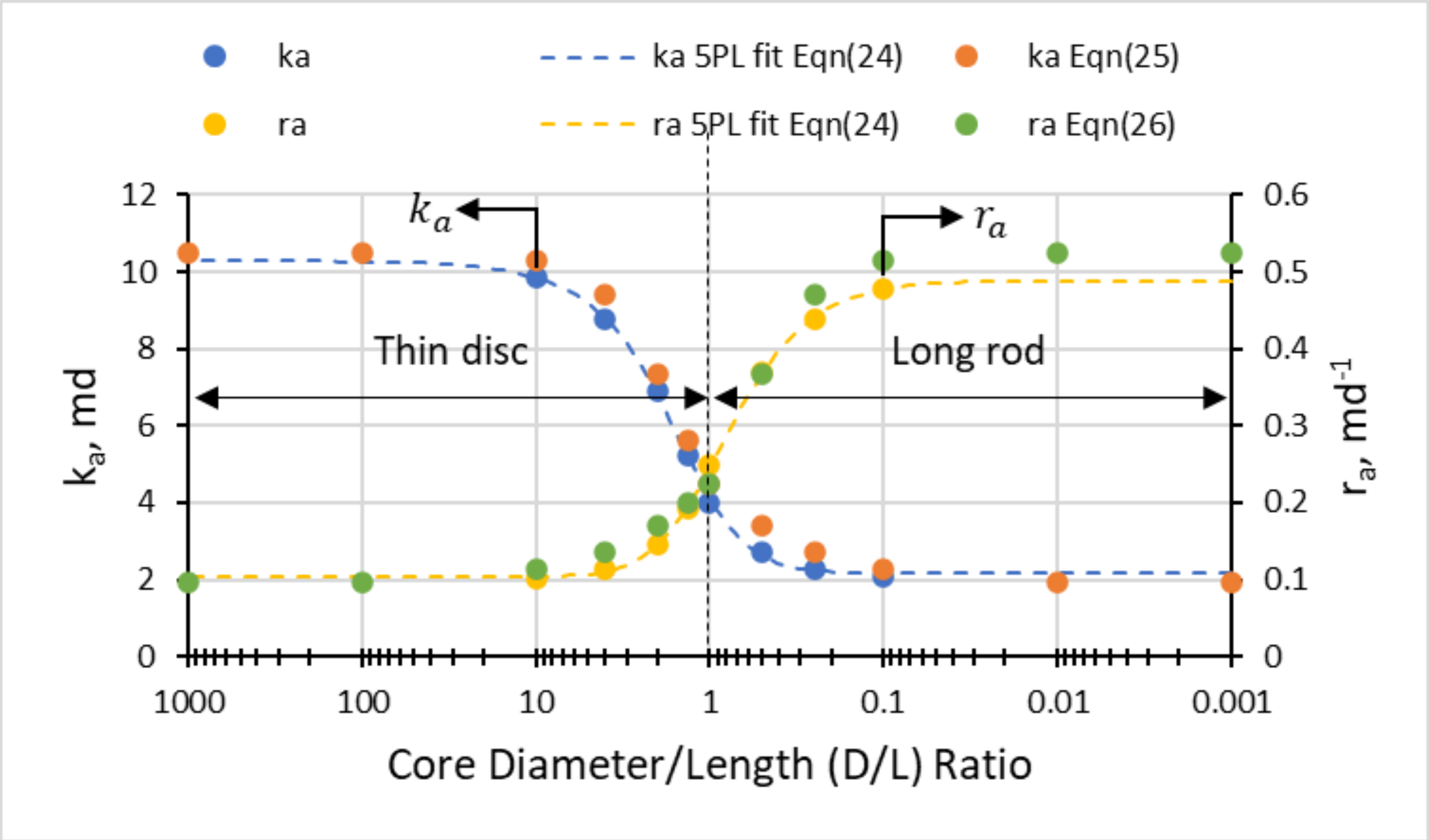


- $k_a = \frac{Q\mu L}{A\Delta\Phi}$
- $r_a = \frac{A\Delta\Phi}{Q\mu L}$
- $y = d + \frac{(a-d)}{\left(1+(x/c)^b\right)^g} \quad (24)$
- $k_a = \sqrt{\frac{k_{ij}u_iu_j}{r_{kl}u_ku_l}\left[\frac{1+\sqrt{k_{ij}u_iu_jr_{kl}u_ku_l}(D/L)^n}{\sqrt{k_{ij}u_iu_jr_{kl}u_ku_l}+(D/L)^n}\right]},$

$$n = \begin{cases} 1, & D/L < 1 \\ 2, & D/L \geq 1 \end{cases} \quad (25)$$

- $r_a = \sqrt{\frac{r_{kl}u_ku_l}{k_{ij}u_iu_j}\left[\frac{\sqrt{k_{ij}u_iu_jr_{kl}u_ku_l}+(D/L)^n}{1+\sqrt{k_{ij}u_iu_jr_{kl}u_ku_l}(D/L)^n}\right]},$

$$n = \begin{cases} 2, & D/L < 1 \\ 1, & D/L \geq 1 \end{cases} \quad (26)$$



Additional results & answers part 1

Q: What is the relation between the geometry of a cylindrical sample, expressed by the D/L ratio, and the apparent directional permeability k_a or apparent directional viscous resistivity r_a for a given anisotropy k_{ij} and measurement orientation u_i?

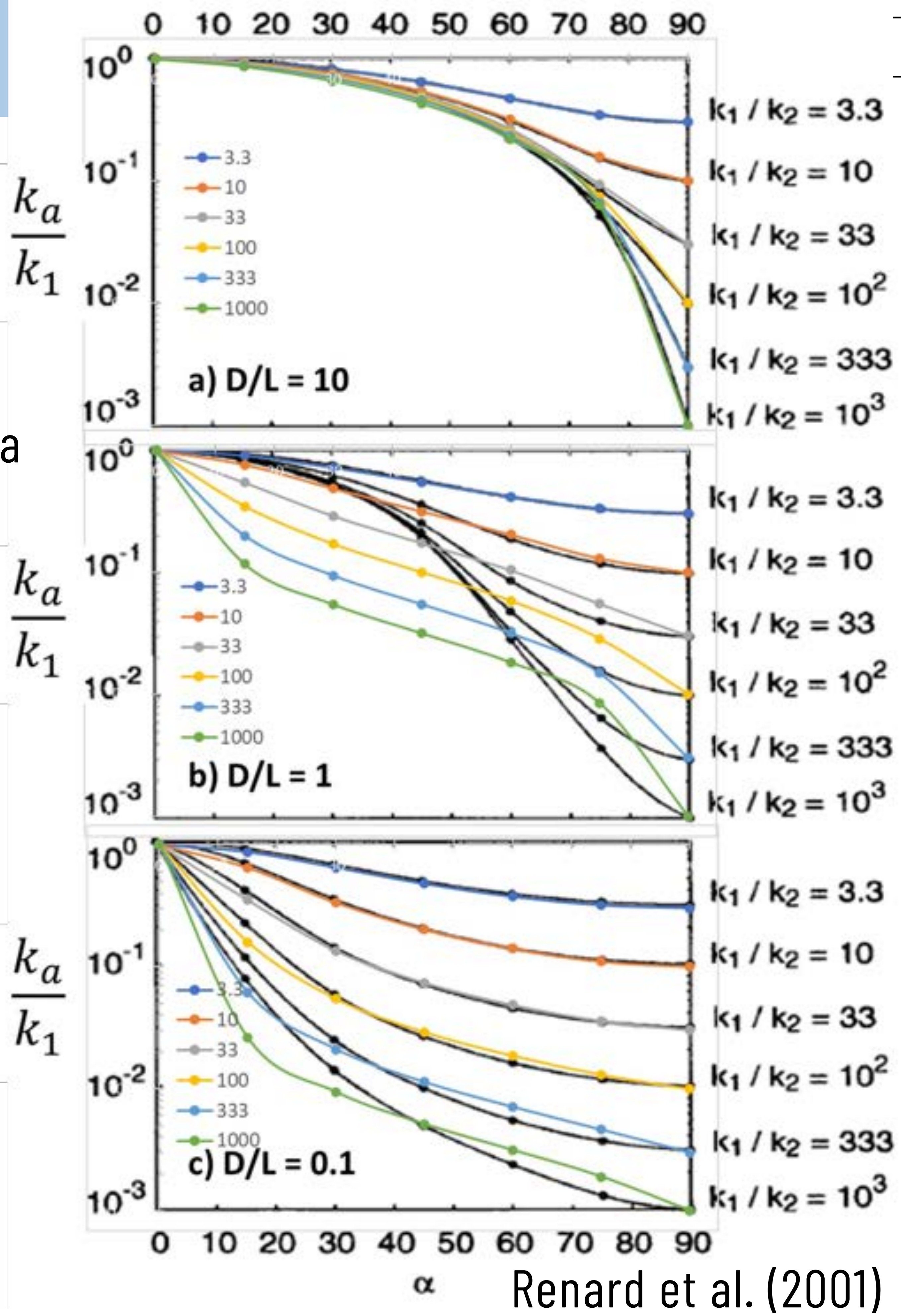
A: To an approximation,

- $$k_a = \sqrt{\frac{k_{ij}u_iu_j}{r_{kl}u_ku_l}\left[\frac{1+\sqrt{k_{ij}u_iu_jr_{kl}u_ku_l}(D/L)^n}{\sqrt{k_{ij}u_iu_jr_{kl}u_ku_l}+(D/L)^n}\right]},$$

$$n = \begin{cases} 1, & D/L < 1 \\ 2, & D/L \geq 1 \end{cases} \quad (25)$$

- $$r_a = \sqrt{\frac{r_{kl}u_ku_l}{k_{ij}u_iu_j}\left[\frac{\sqrt{k_{ij}u_iu_jr_{kl}u_ku_l}+(D/L)^n}{1+\sqrt{k_{ij}u_iu_jr_{kl}u_ku_l}(D/L)^n}\right]},$$

$$n = \begin{cases} 2, & D/L < 1 \\ 1, & D/L \geq 1 \end{cases} \quad (26)$$



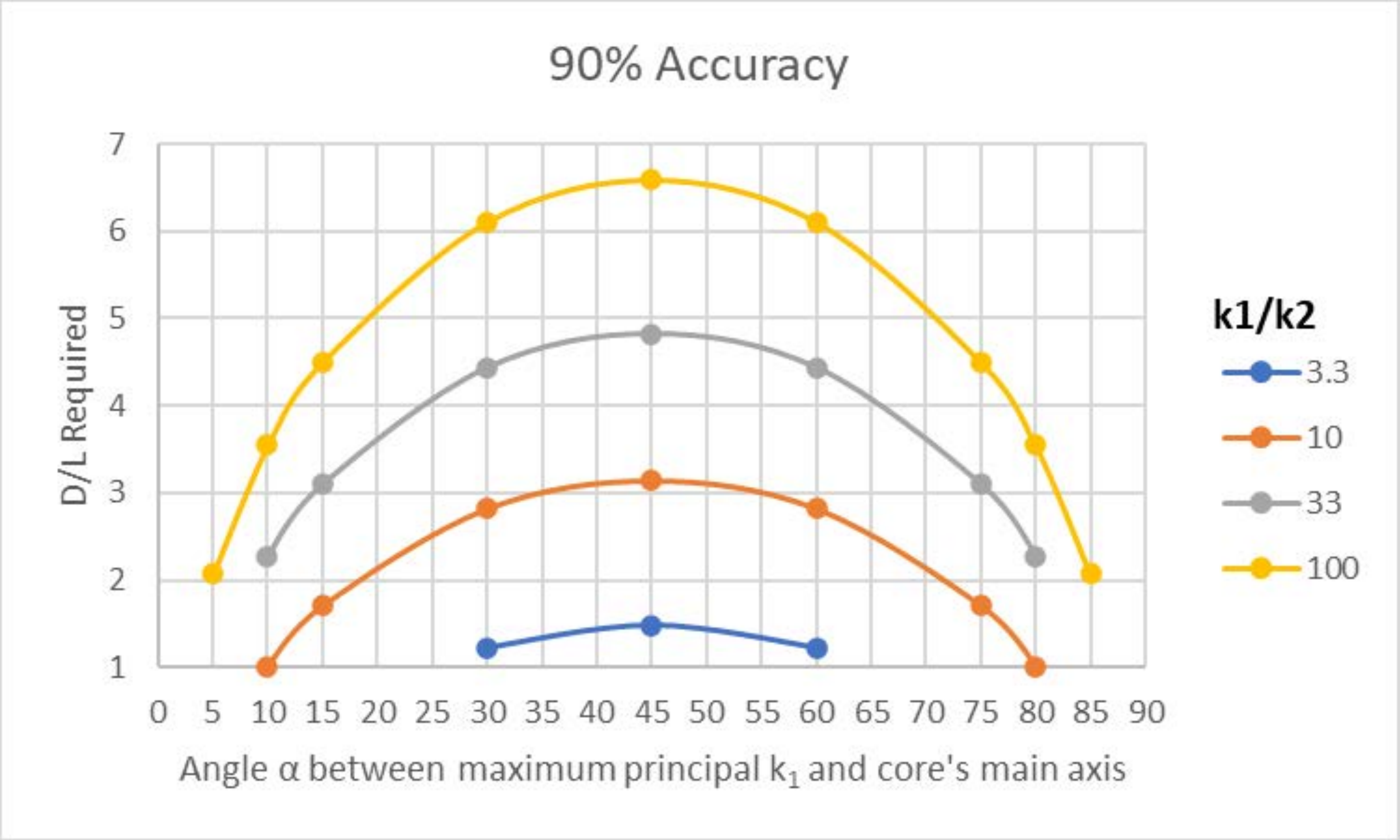
Renard et al. (2001)

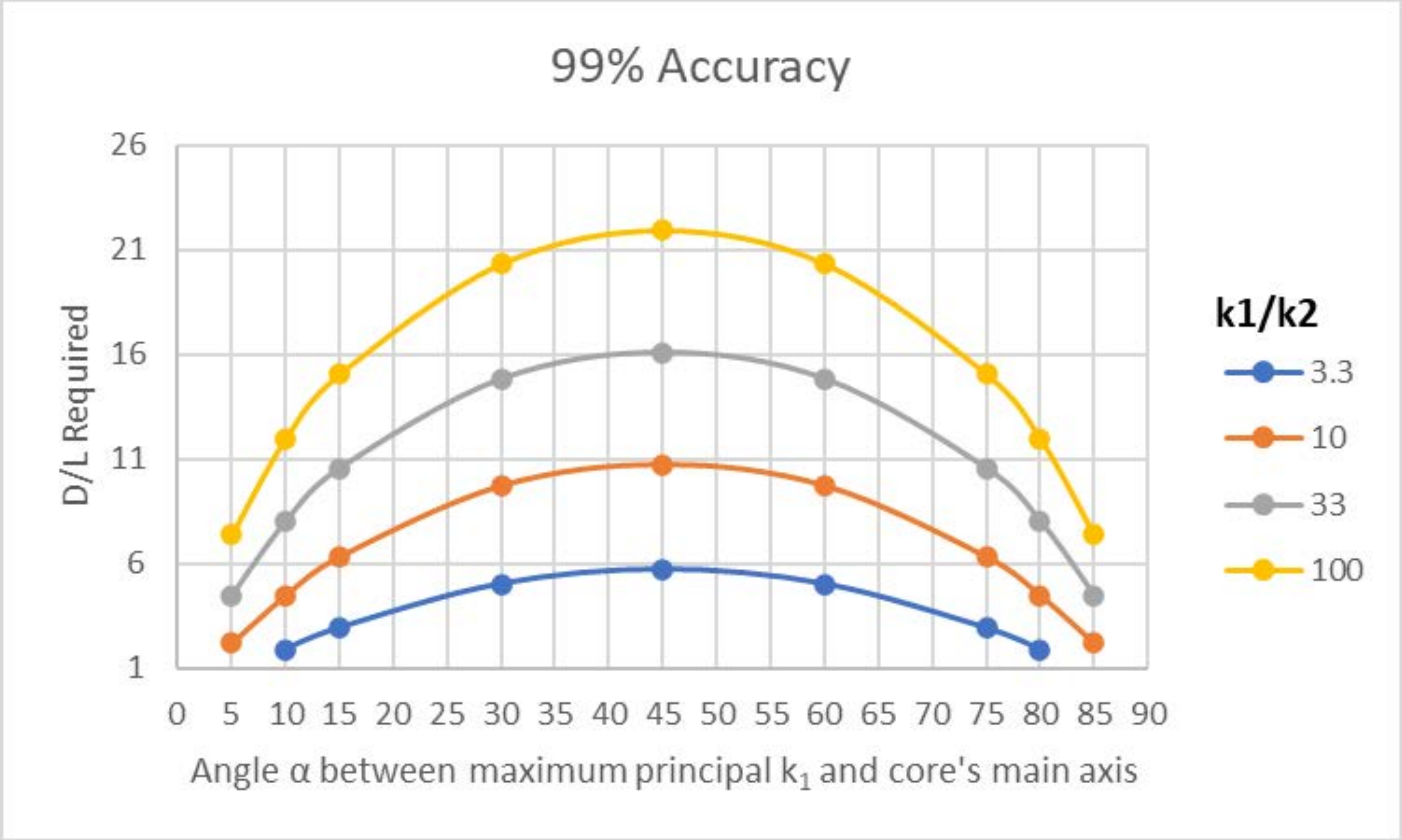
	k_1/k_2	Error (%) Max	Error (%) Avg.
a) D/L = 10	3.3	4	Avg. 4
	10	8	Med.
	33	12	2
	10^2	21	
	333	32	
	10^3	24	
b) D/L = 1	3.3	7	Avg.
	10	21	31
	33	47	Med.
	10^2	77	11
	333	124	
	10^3	128	
c) D/L = 0.1	3.3	2	Avg.
	10	5	11
	33	13	Med.
	10^2	25	3
	333	45	
	10^3	66	

Answers part 2

Q: What dimensions does the sample need to have to yield k or r with a certain desired accuracy?

A: The required dimensions (D/L) is a function of k_{ij} and $u_i(\alpha)$ which can be solved for.

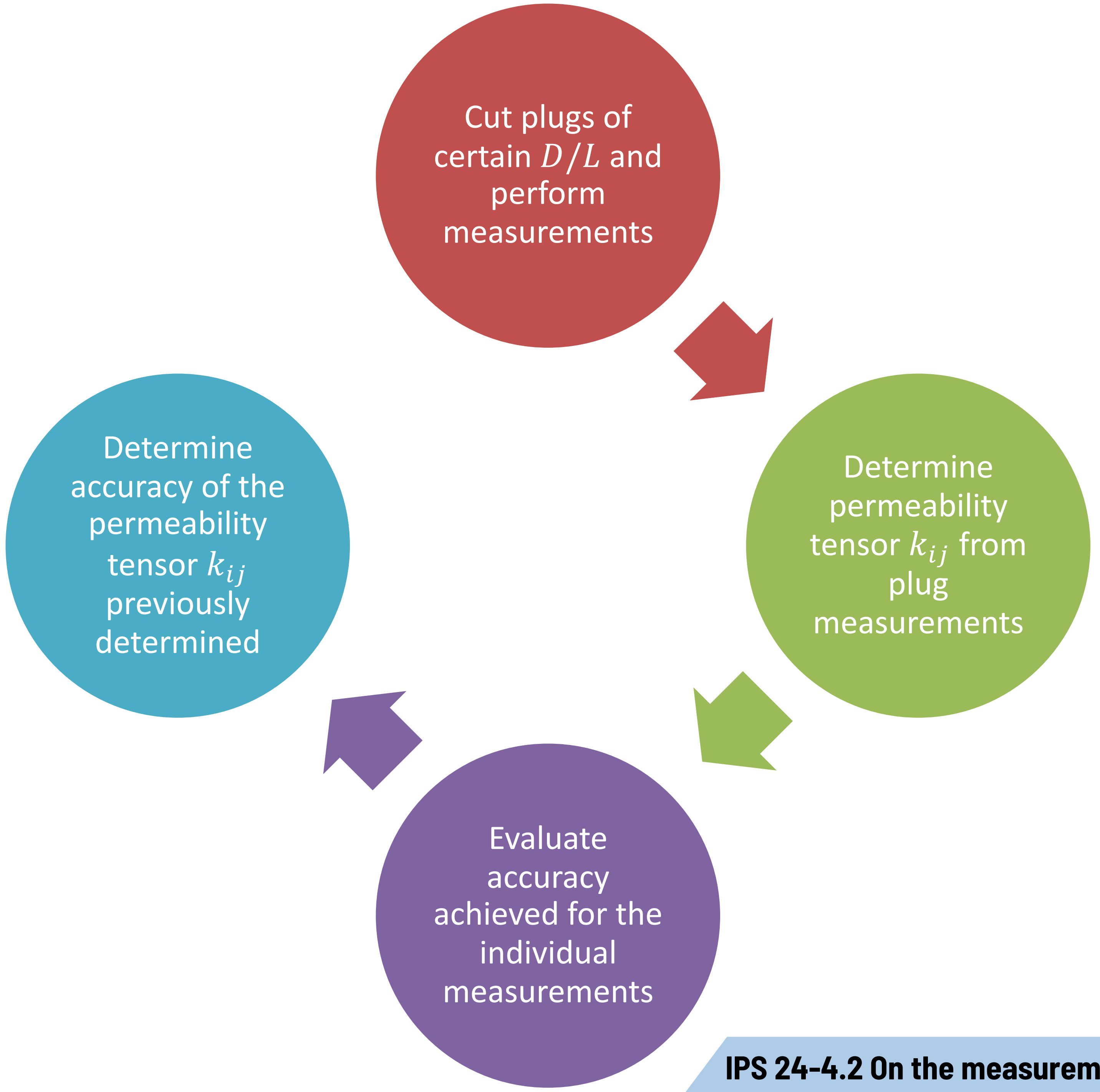




Example 2D anisotropy, required D/L vs desired accuracy plots for directional k as $f(k_1/k_2, \alpha)$.

Analysis of core dimensions for measurement accuracy

An Example Method



Summary

- Accuracy of k_c is dependent on accuracy of k_{virgin} values measured and used in RI_{th} calculation
- For finite-dimensioned samples, k_a and $r_a = f(D/L, k_{ij}, r_{ij}, u_i)$
  - Approximate analytical expressions for k_a and r_a exist and can be used to approximate measurement accuracy
- Thin disc experiment, $D/L \gg 1$ to measure k_a
- Long rod experiment, $D/L \ll 1$ to measure r_a
  - Six or more independent directional measurements are needed to determine the full tensor
    - When one tensor is determined, its reciprocal (matrix inverse) is also known
- A real-world example of experimental data substantiating the 3D tensor theory of viscous resistivity (and permeability) for a geologic porous medium has been presented

For nomenclature and additional info

McGregor, J. (2023). The effect of sample anisotropy properties, dimensions, and imposed no-flow boundaries on the measurement of directional permeability and viscous resistivity. *Symposium of the Society of Core Analysts*. Abu Dhabi, UAE: Society of Core Analysts.

QUESTIONS?

MAY 13-15 IPS 2024

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